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Higher Order Mode Interaction in Nonreciprocal Periodic Structures

T. A. ENEGREN AND M. M. Z. KHARADLY

Abstract—**This paper is an extension of previous work on nonreciprocal periodic structures where only dominant mode interaction was considered. Higher order mode interaction is taken into account by using multimode wave matrix analysis. Numerical results for the propagation coefficient characteristics are given for a specific example of a twin-slab ferrite-loaded rectangular waveguide, periodically loaded with thin metallic diaphragms. These characteristics show similar trends to those observed in a previous experimental investigation.**

I. Introduction

In a recent paper, wave matrix analysis has been used to study nonreciprocal periodic structures [1]. The analysis presented in [1] enabled determination of the propagation constants of the forward and backward traveling Bloch waves, based on single dominant mode interaction between the discontinuities. Comparison between theoretical prediction and measurement for a ferrite loaded waveguide periodically loaded by thin inductive diaphragms showed that the single dominant-mode theory gave good results only when higher order mode interaction between the diaphragms could be neglected [1]. When the spacing between the loading elements became small, there were significant discrepancies between measurement and prediction. This paper shows that these discrepancies can be attributed to the effects of higher order mode interaction between the discontinuities.

The numerical analysis presented in [2] for the scattering problem of the infinitely thin metallic diaphragm in a magnetized ferrite-loaded waveguide can be extended to

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obtain the reflection and transmission coefficients of the higher order modes. These coefficients can then be used in an extended version of the wave matrix analysis to account for the effects of higher order mode interaction. In this paper, this is done for the nonreciprocal periodic structure shown in Fig. 1, which consists of a twin-slab ferrite loaded waveguide, periodically loaded by an array of diaphragms. The numerical results thus obtained confirm the trends found in the experimental investigation of nonreciprocal periodic structures [1].

II. Analysis of Periodic Structure

Consider the periodic structure shown in Fig. 1 consisting of an array of diaphragms in a ferrite-loaded waveguide. Under normal operation, the structure will support a forward and a backward traveling Bloch wave. One method of determining the propagation constants of the Bloch waves uses wave matrix analysis [1], [3]. In this method, the periodic structure is subdivided into unit cells; the equivalent circuit of a cell is shown in Fig. 2(a). The transmission lines represent the electrical length of the unit cell and the three element reactive network represents the equivalent circuit for the diaphragm. Referring to Fig. 2(b), the quantities R_{12}, R_{21} and T_{12}, T_{21} are the reflection and transmission coefficients of the diaphragm for the dominant mode (mode amplitudes given by a_1, b_1 and a_1', b_1'). Thus the wave matrix for the diaphragm is given by

$$\begin{bmatrix} 1/T_{12} & -R_{21}/T_{12} \\ R_{12}/T_{12} & T_{21}-R_{12}R_{21}/T_{12} \end{bmatrix}. \qquad (1)$$

This two-port junction representation of the diaphragm will be adequate as long as the spacing between the diaphragms is large enough to allow the higher order modes excited at one diaphragm to decay to negligible values at the position of the next diaphragm. If this is not the case, however, then an additional equivalent transmission line must be introduced for each mode that appreciably interacts with the adjacent diaphragms [3], as shown in Fig. 3(a). These transmission lines are coupled by the network representing the diaphragm. The approach taken here, for the nonreciprocal case, is essentially an extension of that given in [3] for the reciprocal case.

Let N be the number of interacting modes. The mode amplitudes are related by

$$\begin{aligned} \boldsymbol{b} &= R^{\mathrm{I}}\boldsymbol{a} + T^{\mathrm{II}}\boldsymbol{b}' \\ \boldsymbol{a}' &= R^{\mathrm{II}}\boldsymbol{b}' + T^{\mathrm{I}}\boldsymbol{a} \end{aligned} \qquad (2)$$

where the matrices R^{I}, R^{II} and T^{I}, T^{II} are reflection and transmission coefficient matrices, respectively, and are defined as follows:

- R_{ij}^{α} reflection coefficient for the ith mode with the jth mode incident from waveguide α;
- T_{ij}^{α} transmission coefficient for the ith mode with the jth mode incident from waveguide;
- i, j 1, N;
- α I, II.

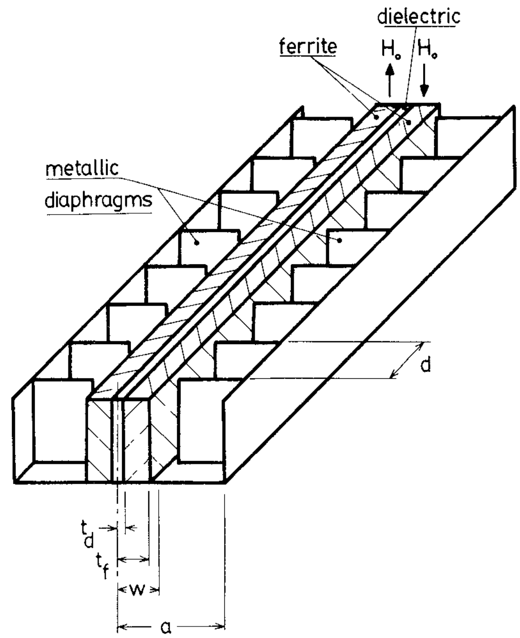

Fig. 1. Periodic loading of theoretical twin-slab configuration of ferrite by "inductive" diaphragms. Frequency = 9 0 GHz, $a = 1$ 143 cm, $t_d = 0.046$ cm, $t_f = 0.255$ cm, $w = 0.352$ cm, relative permittivity of dielectric = 1.0. Ferrite characteristics: relative permittivity = 12.0; relative permeability = 0.96; off diagonal component of permeability tensor = $\pm j0.3$.

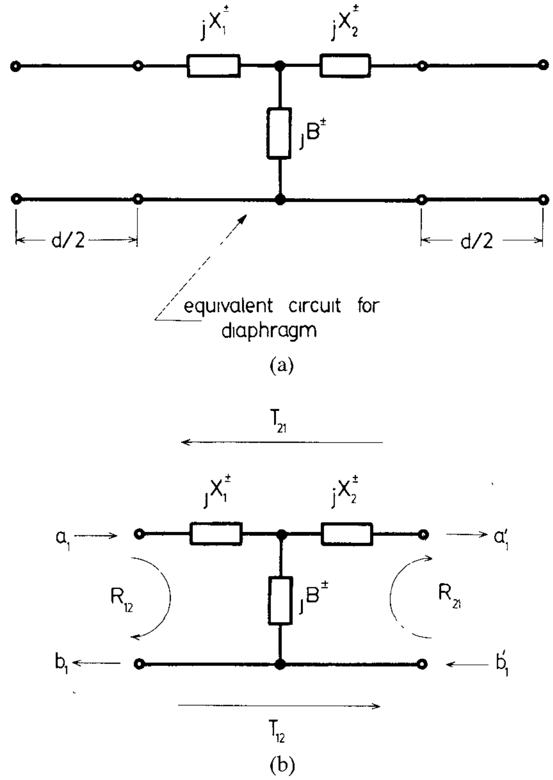

Fig. 2. (a) Equivalent circuit for unit cell. (b) Equivalent circuit for diaphragm.

Values for the matrix elements of R^{I}, R^{II} and T^{I}, T^{II} for an infinitely thin conducting diaphragm may be found using the numerical technique developed in [2]. Briefly, this is done by allowing one mode of unit amplitude to be

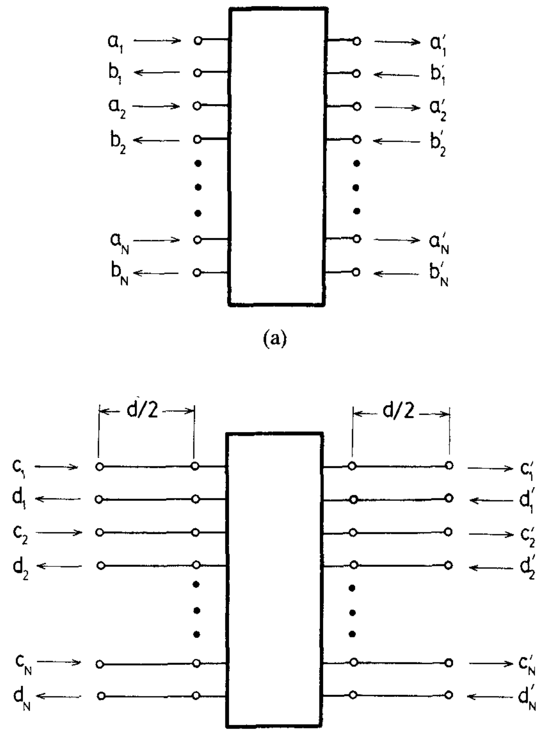

Fig. 3. (a) $2N$ port junction representation of diaphragm. (b) Transmission line representation of unit cell with discontinuity represented by a $2N$ port junction.

incident at the position of the diaphragm and calculating the corresponding reflection and transmission coefficients. This operation is performed in turn for each mode. Equation (2) can be cast into the form

$$\begin{bmatrix} a \\ b \end{bmatrix} = A_d \begin{bmatrix} a' \\ b' \end{bmatrix}. \tag{3}$$

The matrix A_d is the generalized wave matrix for the diaphragm and is given by

$$A_d = \begin{bmatrix} Q & -QR^{\mathrm{II}} \\ R^{\mathrm{I}}Q & T^{\mathrm{II}} - R^{\mathrm{I}}QR^{\mathrm{II}} \end{bmatrix} \tag{4}$$

where $Q = \text{inverse}\,(T^{\mathrm{I}})$.The generalized wave matrix for a section of transmission line of length $d/2$ is given by

$$E^{\mathrm{I}} = \begin{bmatrix} e^{-j\beta_1^+ d/2} & & & \\ & e^{-(\alpha_2^+ + j\beta_2^+)d/2} & & \\ & & \ddots & \\ & & & e^{-(\alpha_N^+ + j\beta_N^+)d/2} \end{bmatrix}$$

$$E^{\mathrm{II}} = \begin{bmatrix} e^{-j\beta_1^- d/2} & & & \\ & e^{-(\alpha_2^- + j\beta_2^-)d/2} & & \\ & & \ddots & \\ & & & e^{-(\alpha_N^- + j\beta_N^-)d/2} \end{bmatrix}. \tag{5}$$

The sign of the propagation constants for the evanescent modes is chosen such that the exponential term decreases

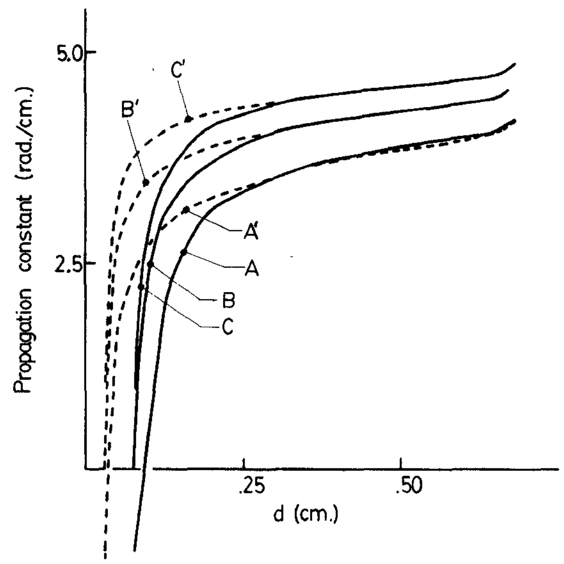

Fig. 4. Variation of propagation constants with spacing d, comparing the analysis based on single-mode interaction with that including two higher order modes: ——— single-mode interaction; – – – higher mode interaction included.

with increasing d. Therefore, the generalized matrix A_{uc} for the unit cell shown in Fig. 3(b) is given by

$$A_{uc} = \begin{bmatrix} \text{inverse}\,(E^{\mathrm{I}}) & 0 \\ 0 & E^{\mathrm{II}} \end{bmatrix} A_d \begin{bmatrix} \text{inverse}\,(E^{\mathrm{I}}) & 0 \\ 0 & E^{\mathrm{II}} \end{bmatrix}. \tag{6}$$

Thus, the mode amplitudes for the unit cell will be related by

$$\begin{bmatrix} c \\ d \end{bmatrix} = A_{uc} \begin{bmatrix} c' \\ d' \end{bmatrix}. \tag{7}$$

The solution for Bloch waves requires that

$$\begin{bmatrix} c \\ d \end{bmatrix} = e^{\gamma d} \begin{bmatrix} c' \\ d' \end{bmatrix}. \tag{8}$$

Equations (7) and (8) lead to the following eigenvalue relation

$$\det\left(A_{uc} - e^{\gamma d} U\right) = 0 \tag{9}$$

where U is the unit matrix. The propagation constant γ of any particular Bloch wave may be determined from (9). There are $2N$ solutions to this equation, corresponding to N modes traveling in the forward direction and N modes traveling in the backward direction. In practice, only one mode is allowed to propagate in each direction.

The implementation of the above analysis is illustrated by considering a numerical example. In this example, with the dimensions shown in Fig. 1 and the frequency set at 9.0 GHz, the width of the diaphragm was chosen so that the first two evanescent modes were excited much more significantly than the third- and higher order modes. Thus only the first two evanescent modes were considered and this limited the size of the reflection and transmission coefficient matrices to dimension 3.

The propagation constants for the propagating Bloch waves are plotted in Fig. 4 against the spacing between the diaphragms. The curves A, A' and C, C' are for the case

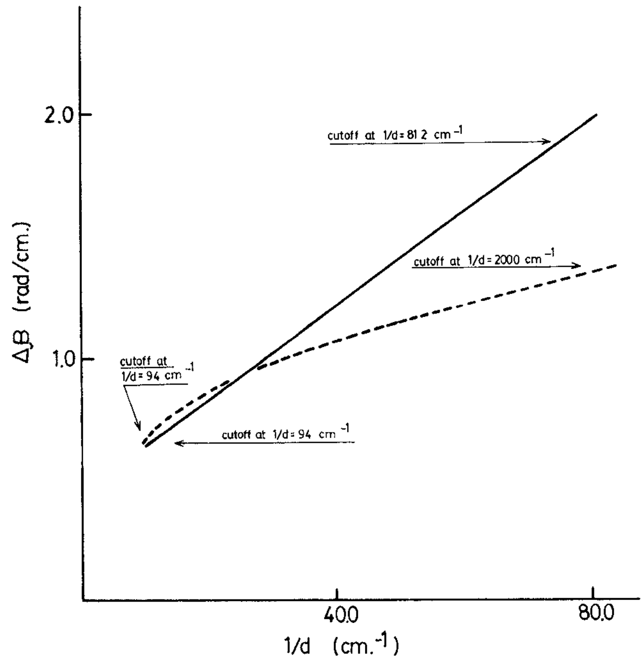

Fig. 5. Variation of $\Delta\beta$ with $1/d$, comparing the analysis based on single-mode and that including higher mode interaction: ——— single-mode interaction; – – – –higher mode interaction included.

where the ferrite slabs are magnetized (A and A' are for the magnetizing sense of Fig. 1, C and C' for the opposite sense). The curves B and B' are for the case where the ferrite slabs are unmagnetized. The results of the analysis based on only dominant mode interaction is indicated by the solid lines. The effect of the inclusion of higher mode interaction is indicated by the dashed lines.

These results show two main characteristics: 1) that the propagation constants are generally higher than those predicted using dominant mode interaction only; and 2) that the spacing of the periodic structure can be made smaller before cutoff occurs. This is basically what has been observed in the experimental investigation of [1]. A comparison of the differential phase-shift characteristics is shown in Fig. 5. Generally, the differential phase shift per unit length is reduced when the effects of higher mode interaction are included. This also confirms the experimental findings in [1].

III. Conclusions

A numerical technique has been presented to account for the effects of higher mode interaction nonreciprocal periodic structures. This involves expanding the wave transmission matrix for the discontinuity to include those higher order modes which significantly interact. The reflection and transmission coefficients for these modes are found using the mode-matching technique. The numerical results confirm the trends previously observed in an experimental investigation.

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